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ADDENDUM TO SIMPLIFIED MATHEMATICAL MODELING OF WATER QUALITY

Environmental Protection Agency
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WATER QUALITY MANAGEMENT PLANNING

Addendum to Simplified Mathematical Modeling of Water Quality

THIS REPORT SUPPORTS MATERIAL PRESENTED IN THE MAIN TEXT, "SIMPLIFIED MATHEMATICAL MODELING OF WATER QUALITY" AS THIS ADDENDUM EXPLAINS IN DETAIL SOME OF THE SIMLIFYING ASSUMPTIONS MADE IN THE MODEL DEVELOPMENT, IT IS STRONGLY RECOMMENDED THAT USERS CAREFULLY REVIEW THIS DOCUMENT BEFORE ATTEMPTING TO APPLY THE MODELS PRESENTED IN THE MAIN TEXT.

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INTRODUCTION

This Addendum was prepared in response to suggestions from EPA regional personnel on topics needing additional comment or clarification in connection with the handbook, "Simplified Mathematical Modeling of Water Quality". Topics covered in this Addendum are:

Section	Title	Page No.
I	Waste Treatment Efficiencies	E-1
II	Nitrification	E-9
III	Determination of $\emptyset$ from Observed Data	E-15
IV	Reaeration Over Dams	E-25
V	Temperature Coefficients (for K_d, K_a)	E-35
VI	Evaluation of the Dispersion Coefficient, E	E-35

Sections I and II, which explain some of the simplifying assumptions made during model development, indicate specific situations where additional discretion and judgment may be required before using the handbook. When lagoon treatment is being considered, Figure E-1A from Section I replaces Figure B-1 (Chart B) and Figure E-1 (Chart E) from the main text. If the possibility of high algal concentrations in lagoon effluent exists, Section I recommends against the use of this guide. All handbook users faced with possible water quality problems related to nitrogenous oxgen demand should read Section II which discusses the assumptions made in the main text concerning nitrification. Sections III, V, and VI provide additional assistance on the derivation and use of several model coefficients. Section IV presents new material on modeling reaeration over dams; this topic is not discussed in the main text.

An errata sheet for the main text is included at the end of this Addendum.

APPENDIX E
ADDENDUM

I. Waste Treatment Efficiencies

The purpose of this section is to further explain the bases of Tables III-3 and III-4 of the main text (see pages 53-54), which deal with the estimated efficiencies of treatment levels. The basic assumptions used for these tables were: (a) 125 gallons/capita-day; (b) 0.174 pounds five-day carbonaceous BOD ($CBOD_5$)/capita-day; (c) 0.044 pounds oxidizable nitrogen (organic nitrogen and ammonia)/capita-day; and (d) 0.025 pounds PO_4/capita-day. The ultimate to five-day carbonaceous BOD ratio used was 1.43. This term results from a BOD bottle decay coefficient of 0.24/day, which is representative of municipal wastes. The ultimate nitrogenous oxygen demand to nitrogen ratio used was 4.57. This term results from the stoichiometric balance of the conversion of ammonia to nitrate, which shall be discussed in Section II of this addendum. Application of these ratios resulted in (a) 0.25 pounds ultimate carbonaceous BOD/capita-day and (b) 0.20 pounds ultimate nitrogenous BOD/capita-day.

With respect to carbonaceous BOD, the column labeled "% Removal" in Table III-3 is in terms of estimated $CBOD_5$ removal efficiencies. Since carbonaceous removal efficiencies are usually reported in terms of $CBOD_5$, the normal procedure in a stream or estuary analysis is to calculate the pounds of $CBOD_5$ inputed into the water body, and subsequently increase this load by the ultimate to five-day carbonaceous BOD ratio. In Table III-3, the calculations were initiated with the 0.25 pounds ultimate carbonaceous BOD/capita-day, and the $CBOD_5$ efficiencies were applied to this value based on the assumption that the ultimate to five-day carbonaceous BOD ratio does not significantly change in the secondary effluent. Under this assumption, both procedures result in the same value when reduced to the ultimate carbonaceous demand inputed into the water body. The ultimate to five-day carbonaceous BOD ratio may increase with more advanced treatment. However, the value used is appropriate for interim planning.

Figures B-1 (Chart B) and E-1 (Chart E) of the main text are obtained directly from Table III-3. The following illustrative example problem on treatment efficiencies is presented:

Illustrative Example

Existing Population - 25,500

Proposed Treatment - High Rate Biological Treatment

DO Standard - 4 mg/l

River - Intermediate Channel, depth 5-10 feet

Drainage Area at Discharge Location - 1,700 mi^2

Estimated Drought Flow Rate - .05 cfs/mi^2

Maximum Water Temperature - 30°C

1) From knowledge of Area - moderate to high growth
   f_1 = 2.00 (Table III-1, p. 49)
   Design Popultion: 25,000 x 2.00 = 51,000

2) Chart B - Enter Figure B-1
   Design Popultion - 51,000 at High Rate Treatment gives effluent load of 10,000 lb/day ultimate BOD.
   Calculation:

   HR-BIO Treatment - Table III-3
   85% removal of $CBOD_5$
   20% removal of oxidizable N

$$0.25 \frac{\text{lbs C - UOD}}{\text{cap-day}} \times (1 - .85) = .037 \frac{\text{lbs C - UOD}}{\text{cap-day}}$$

$$0.20 \frac{\text{lbs N - UOD}}{\text{cap-day}} \times (1 - .20) = .160 \frac{\text{lbs N - UOD}}{\text{cap-day}}$$

$$\text{TOTAL} = .197 \frac{\text{lbs - UOD}}{\text{cap-day}}$$

$.197 \times 51{,}000 = 10{,}000$ lb UOD/day

Note: $\frac{.197}{.25 + .20}(100) = 44\%$ Remaining

56% Removal of UOD

Estimation of Municipal Loads

$$0.25 \frac{\text{lbs C - UOD}}{\text{cap-day}} / 1.43 = 0.174 \frac{\text{lbs CBOD}_5}{\text{cap-day}}$$

$$0.20 \frac{\text{lbs N - UOD}}{\text{cap-day}} / 4.57 = 0.044 \frac{\text{lbs oxid. N}}{\text{cap-day}}$$

at 125 gallons/capita-day

$$\text{CBOD}_5 = \frac{.174 \text{ lbs CBOD}_5\text{/cap-day}}{125 \text{ gal/cap-day}} \times \frac{10^6 \text{ mg/l}}{8.34 \text{ lbs/gal}}$$

$$\simeq 167 \text{ mg/l}$$

$$\text{Oxid.N} = \frac{.044 \text{ lbs Oxid.N/cap-day}}{125} \times \frac{10^6}{8.34}$$

$$= 42 \text{ mg/l}$$

3) Alternate Calculation:

Effluent CBOD_5 reported in design as 25 mg/l

$$\text{Flow} = \frac{51{,}000 \times 125 \text{ gallons/cap-day}}{10^6 \text{ gal/MG}}$$

$$= 6.375 \text{ MGD}$$

No nitrogen information.
Assume 20% removal of Oxid. N

$$42 \text{ mg/l-N} \times (1 - .20) \times 4.57 \frac{\text{lbs N - UOD}}{\text{lbs Oxid. N}}$$

$$\times 6.375 \text{ MGD} \times 8.34 \frac{\text{lbs/MG}}{\text{mg/l}} = 8160 \text{ lbs N-UOD/day}$$

$$25 \text{ mg/l } CBOD_5 \times 1.43 \frac{\text{lbs C - UOD}}{\text{lbs C } BOD_5} \times 6.375 \times 8.34$$

= 1900 lbs C-UOD/day

Total ≈ 10,000 lbs UOD/day

4) See page 86 for solution

Allowable load - 4,000 lbs UOD/day

5) Review of existing data indicates 20 mg/l Oxid N in effluent

20 mg/l-N x 4.57 x 6.375 x 8.34 = 4860 lbs N-UOD/day

1900 lbs C-UOD/day

TOTAL = 6760 lbs UOD/day

Additional Treatment still required to meet DO standard in the future.

6) The planner may investigate when additional treatment is required. At the present poplation the load is 3380 lbs UOD/day. Since the allowable load is 4000 lbs UOD/day, the DO standard will be met under the proposed treatment scheme for the present. Therefore the planner could allow construction of the HR-BIO system now with provisions for the additional treatment needed as growth occurs. A detailed planning study would be required after construction.

Where sufficient land area is available lagoons have been employed as a method of treatment. Lagoons may be classified as four types:

Type I - Aerobic Algae Ponds } "Stabilization" basins
Type II - Facultative Ponds } or simple lagoons
Type III - Anaerobic Ponds }

Type IV - Aerated lagoons - mechanical aeration

As a whole, lagoon treatment efficiencies average about 80% $CBOD_5$ removal, with a range of 50% to 90%. In the colder climates, the lower efficiencies will be encountered during the winter, due to the sensitivity of this type of treatment to ambient temperatures. In mechanically aerated lagoons, nitrogen (organic nitrogen and ammonia) removals may range from 10% to 20%. However, for "stabilization" basins, or simple lagoons which are not mechanically aerated, an overall average for nitrogen removals cannot be realistically defined, due to the seasonal dependence of these forms of treatment with the associated wide variation in nitrogen removals. This situation is further complicated in that the effluents from aerobic algae ponds and facultative ponds often have a high content of organic nitrogen. This nitrogen is associated with both living cells and detritus. The susceptability to hydrolysis and the rate of oxidation will vary widely depending on the form of organic nitrogen in the effluent.

For the purposes of the report, when lagoon treatment is considered, Figure E-1A replaces Figure B-1 (Chart

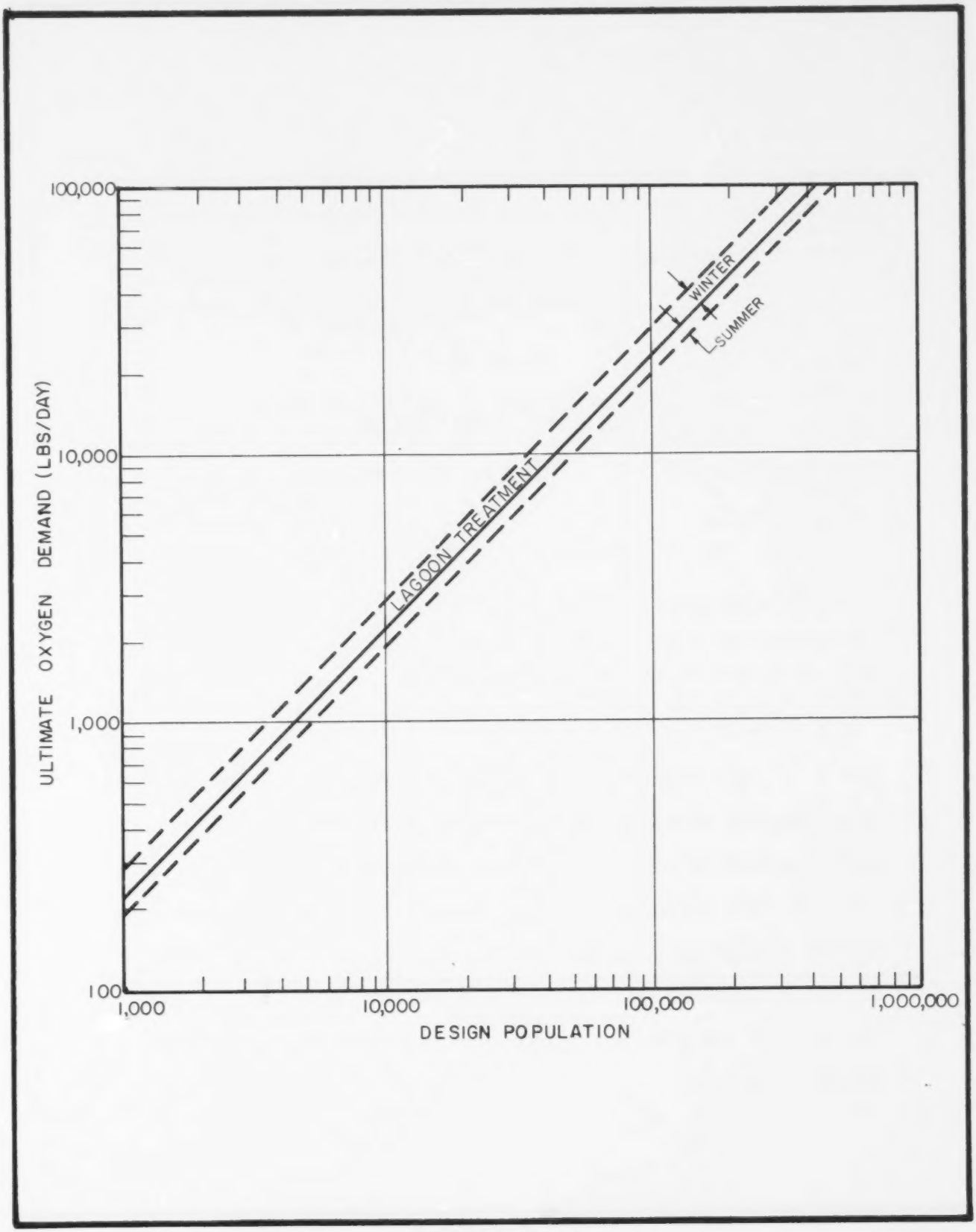

FIGURE E-1A

LAGOON TREATMENT EFFICIENCIES

B), and Figure E-1 (Chart E). The discharged loads shown in Figure E-1A make use of the following table:

TABLE E-1

ESTIMATED EFFICIENCY OF LAGOON TREATMENT
ULTIMATE OXYGEN DEMAND

Treatment Level	% Removal		#/capita/UOD remaining			f_4
	C^*	N^+	C^*	N^+	total	fraction UOD remaining
Lagoon	80	15	.050	.170	.220	.49

C^* = carbonaceous BOD
N^+ = oxidizable nitrogen
f_4 = residual fraction after treatment

The separation of algae and other suspended matter from the effluents of aerobic algae ponds and facultative ponds generally result in higher nitrogen removals than indicated in the foregoing table. However, higher nitrogen removals should only be applied when justified on the basis of consistent historical data of removal efficiencies of "stabilization" basins in the specific geographical area under investigation.

When there is no separation, the algae in the effluents of aerobic algae ponds and facultative ponds may cause significant diurnal variations in dissolved oxygen in a stream. In such a situation, the modeling procedures presented in this guide are inadequate. In general, when the possibility of high algal concentrations in an effluent exist, the use of this guide is not recommended. In addition the possible high solids content and color of such effluents may be of concern.

II. Nitrification

In addition to the oxidation of carbonaceous material in a water body, the oxidation of nitrogen by specific bacteria also exerts an oxygen demand on the water body. In long term biochemical oxygen demand tests, this is usually observed as a second stage. This phenomenon is called nitrification. The important forms of nitrogen in this phenomenon are:

(a) organic nitrogen (amines, proteins)

(b) ammonia

(c) nitrite

(d) nitrate

All forms may be discharged by municipal and industrial waste sources. Organic and ammonia nitrogen, however, are the more common forms discharged by municipal sources. A series of reactions takes place in the nitrification phenomenon which essentially convert organic and ammonia nitrogen into nitrate. In the process of this bacterial conversion or oxidation, oxygen is drawn from available resources to allow the reaction to proceed. Figure II-1 of the main text (see page 20), shows this process.

Organic nitrogen, when discharged into a water body, undergoes an hydrolysis reaction with ammonia as one of the end products. The ammonia formed from organic nitrogen, together with direct discharges of ammonia from waste sources, is oxidized under aerobic conditions to nitrite by bacteria of the genus Nitrosomonas, as follows[1,2,3]:

$$(NH_4) + OH^- + 1.5\ O_2 \xrightarrow{\text{bacteria}} H^+ + NO_2^- + 2H_2O + 59.4\ \text{Kcal.}$$

Note that oxygen is utilized in this biochemical reaction. Stoichiometrically, the reaction requires 3.43 pounds of oxygen for one pound of nitrogen oxidized to nitrite.

The nitrite formed is subsequently oxidized to nitrate by the Nitrobacter as follows[3]:

$$NO_2^- + 0.5\ O_2 \xrightarrow{\text{bacteria}} NO_3^- + 18\ \text{Kcal.}$$

This reaction requires 1.14 pounds of oxygen for one pound of nitrite nitrogen oxidized to nitrate. The total oxygen utilization in the entire forward nitrification process is 4.57 pounds of oxygen per pound of ammonia nitrogen. By contrast, carbonaceous BOD_5 has an ultimate oxygen demand of about 1.43. Thus, a pound of carbonaceous BOD_5 requires about 1.43 pounds of oxygen, while a pound of organic nitrogen or ammonia requires 4.57 pounds of oxygen. Nitrification reactions are, therefore, a potential large source of oxygen depletion in natural waters.

The forward sequential nitrification phenomenon is illustrated in Figure E-2. If the sequence of reactions is allowed to proceed uninterrupted, the ammonia decays almost exponentially, nitrite builds up, but is quickly oxidized to nitrate. Eventually, all of the original ammonia nitrogen is converted bacteriologically to the nitrate form of nitrogen, a bacterially stable end product. In natural waters subjected to large discharges of nitrogenous waste materials, the reaction generally proceeds in the direction indicated in Figure E-2. There are several factors which may modify the sequence of the reactions. Under conditions of low dissolved

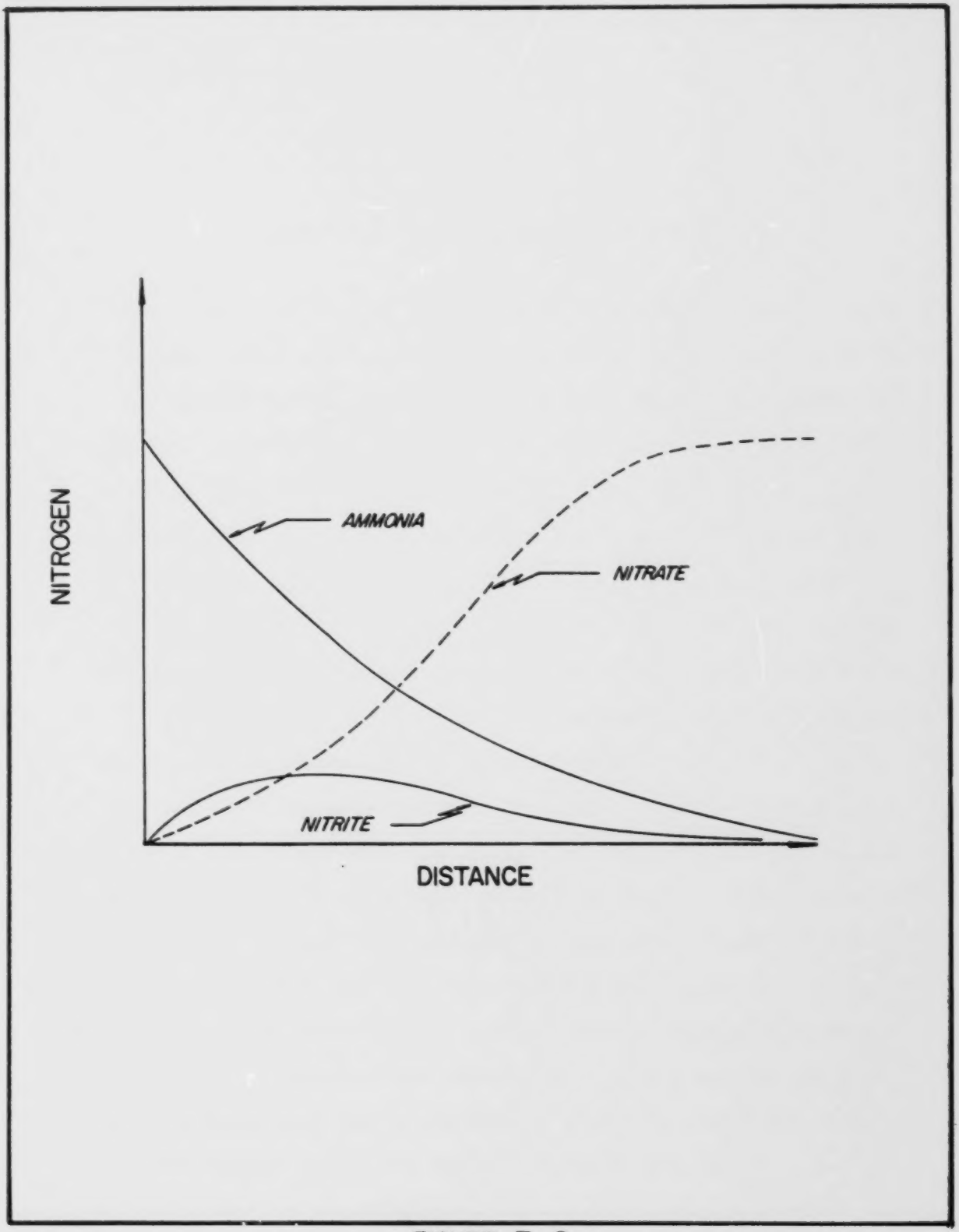

FIGURE E-2
SEQUENTIAL REACTIONS IN NITRIFICATION

oxygen, nitrification is inhibited, and at values approaching zero, nitrification may be completely suppressed. This has been experimentally verified by the British in the Thames Estuary work[4], and earlier by Waksman[5]. For all practical purposes, nitrification may be assumed suppressed at dissolved oxygen levels below 1.5 mg/l. However, as a water body recovers, nitrification may resume and cause a secondary dissolved oxygen sag further downstream. Since dissolved oxygen concentrations of less than 1.5 mg/l are intolerable in most cases, the modeling of the secondary sag has not been included in this guide. This may, however, cause some difficulty in verification analyses, as shall be discussed in Section III of the addendum.

At low water temperatures, less than 10°C, nitrification is inhibited[6]. In application of this guide, it may be assumed that nitrification is suppressed at water temperatures below 10°C. The suppression of nitrification can be simulated by eliminating the nitrogenous ultimate oxygen demand portion of the waste input in the modeling analyses. This concept is important in the evaluation of seasonal treatment schemes.

If significant quantities of phytoplankton are present, ammonia will be used as a nutrient source. As a result,

the available ammonia for nitrification is depleted. In this situation, the diurnal variation in dissolved oxygen caused by the phytoplankton may create problems of low dissolved oxygen at times.

Sawyer[7] indicates that the optimum pH range for nitrification is 8.0 to 8.5. An optimum pH range of 7.0 to 9.0 was reported by Engle and Alexander[8]. Nitrification studies performed by Hydroscience, Inc., indicate that optimum nitrification is not significantly affected within a pH range of 7.0 to 8.5. Since most natural waters are within these ranges, pH should normally not hinder nitrification.

Nitrifying bacteria use free carbon dioxide or bicarbonate ions[4] as the primary sources of carbon for the growth of new cells. Generally, in natural waters, the concentration of inorganic carbon available in the above forms is greatly in excess of requirements and nitrification proceeds.

Under conditions of high organic carbon concentrations, the heterotrophic bacteria (organic carbon oxidizers) may predominate over the autotrophic bacteria (nitrogen oxidizers) which would result in a delay in nitrification. This is usually exemplified in the standard biochemical oxygen demand test by the occurence of two stages. With untreated and

heavily polluted water, the two stages are usually distinct. The first stage, which reflects the aerobic oxidation of the organic material, is usually substantially completed before the second stage, which reflects the oxidation of ammonia through nitrite to nitrate, is significantly underway. In treated effluents and less polluted waters, the lag between the two stages may be reduced. As the first stage is reduced with nitrifying organisms present, the two stages may occur simultaneously. Since this guide will be applied to effluents which have at least secondary treatment, nitrification has been assumed to occur simultaneously with the oxidation of carbonaceous material in the modeling procedures.

III. Determination of ϕ from Observed Data

The deoxygenation coefficient, K_d, and the reaeration coefficient, K_a, can be estimated from field data. With both stream area and flow as constants (an underlying assumption used in the guide), Equation (IV-2) of the main text (see page 69), yields:

$$L = L_o e^{-\frac{K_d x}{U}} \qquad \text{(E-1)}$$

where:

L = BOD concentration (mg/l)

L_o = BOD concentration at x = 0 (mg/l)

U = velocity (miles/day)

K_d = deoxygenation coefficient (1/day)

x = distance (miles)

The field estimate at K_d can be obtained from semi-logarithmic plots of observed long term ultimate BOD stream data (five day BOD stream data may also be used) as a function of distance downstream. Thus, in natural logs, Equation (E-1) is written as:

$$\ln L = - \frac{K_d x}{U} + \ln L_o \qquad \text{(E-1a)}$$

A semi-log plot of field data usually results in a straight line, the slope of which is defined by:

$$\text{Slope} = - \frac{K_d}{U} \qquad \text{(E-1b)}$$

This procedure provides a first estimate of the deoxygenation coefficient. An example of this procedure is shown in Figure E-3 where a straight line has been fitted by eye to the Mohawk

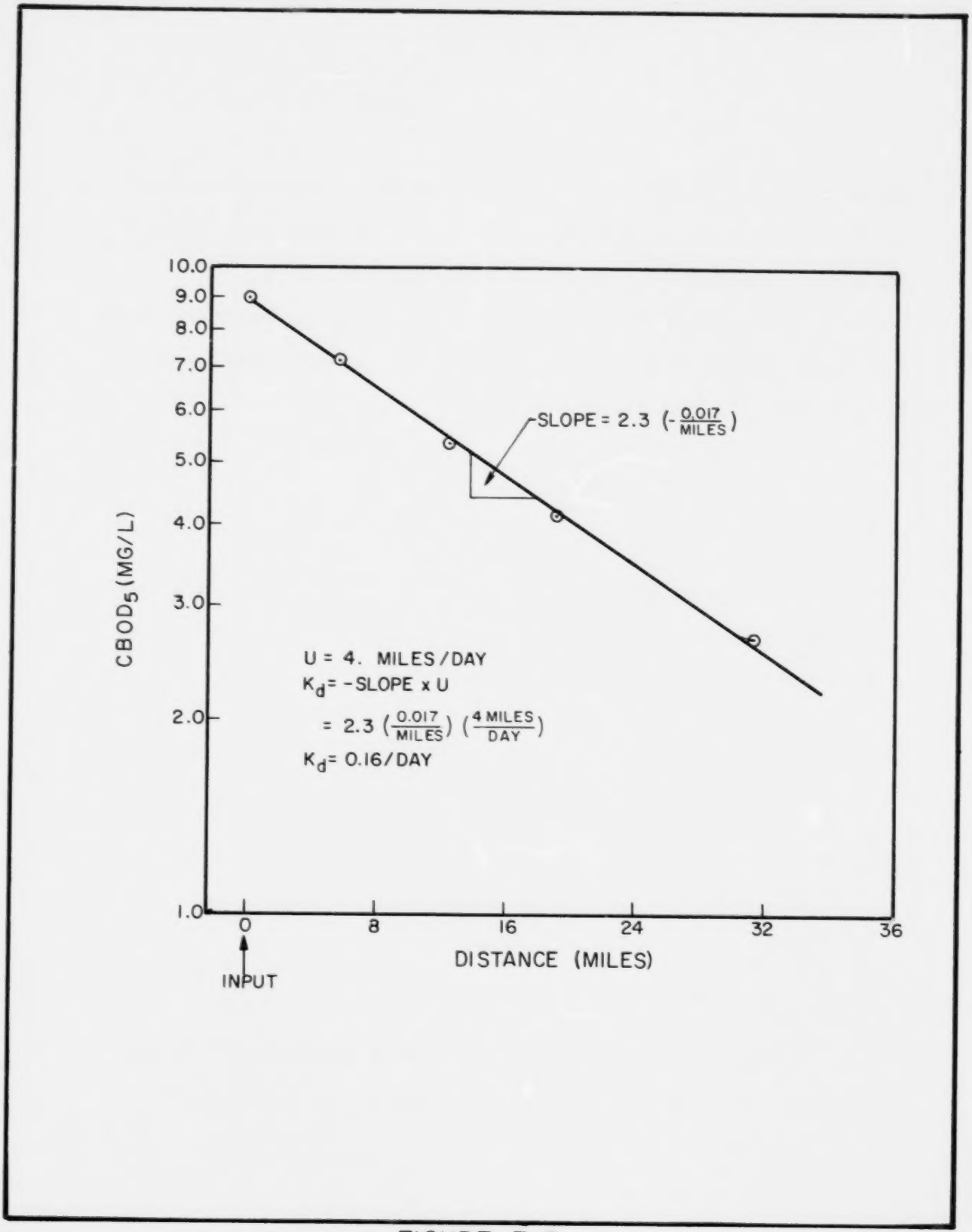

FIGURE E-3

ILLUSTRATION OF COMPUTATION OF K_d
FROM BOD STREAM DATA

River data. Note that care must be taken to include the conversion from base 10 logarithms as given in semi-log plots to base e logarithms as required by Equation (E-1).

The reaeration coefficient, K_a, may be estimated from depth and velocity data by the following formula developed by O'Connor[9] from field studies:

$$K_a = \frac{(D_L U)^{1/2}}{H^{3/2}} \qquad \text{(E-2)}$$

where D_L is the oxygen diffusivity at 20°C (0.000081 ft^2/hour) U is the average stream velocity, and H is the average depth. A more convenient from of this equation is:

$$K_a = \frac{12.9U^{1/2}}{H^{3/2}} \qquad \text{(E-2a)}$$

where:

K_a = reaeration coefficient (1/day)

U = velocity (fps)

H = depth (feet)

The reaeration coefficient is a surface controlled phenomenon and for all practical purposes, is independent of

water quality. In view of this, K_d is usually the more variable unknown coefficient in the term ϕ. Therefore, in estimating ϕ, it is necessary that the K_a and K_d terms be investigated separately as indicated.

The estimation of the reaction coefficients K_d and K_a from which ϕ is obtained along with the estimation of the loading conditions from observed data is the first step in a verification analysis. The general procedure for a verification analysis is presented in Section II-F of the main text (see page 22). Some typical dissolved oxygen verifications accomplished by Hydroscience, Inc.[10, 11] are shown in Figures E-4 and E-5.

There are, however, possible difficulties or "pitfalls" that may be encountered in the verification analysis using the basic assumptions applied in this guide.

For example, assume that a verification is to be attempted in a stream into which a raw municipal effluent is presently inputed. Such an effluent probably has highly settleable solids. As a result, in the vicinity of the outfall, the removal of ultimate oxygen demand is accomplished by the physical settling and the oxidation of the organic matter, simultaneously. Oxidation of the ultimate oxygen

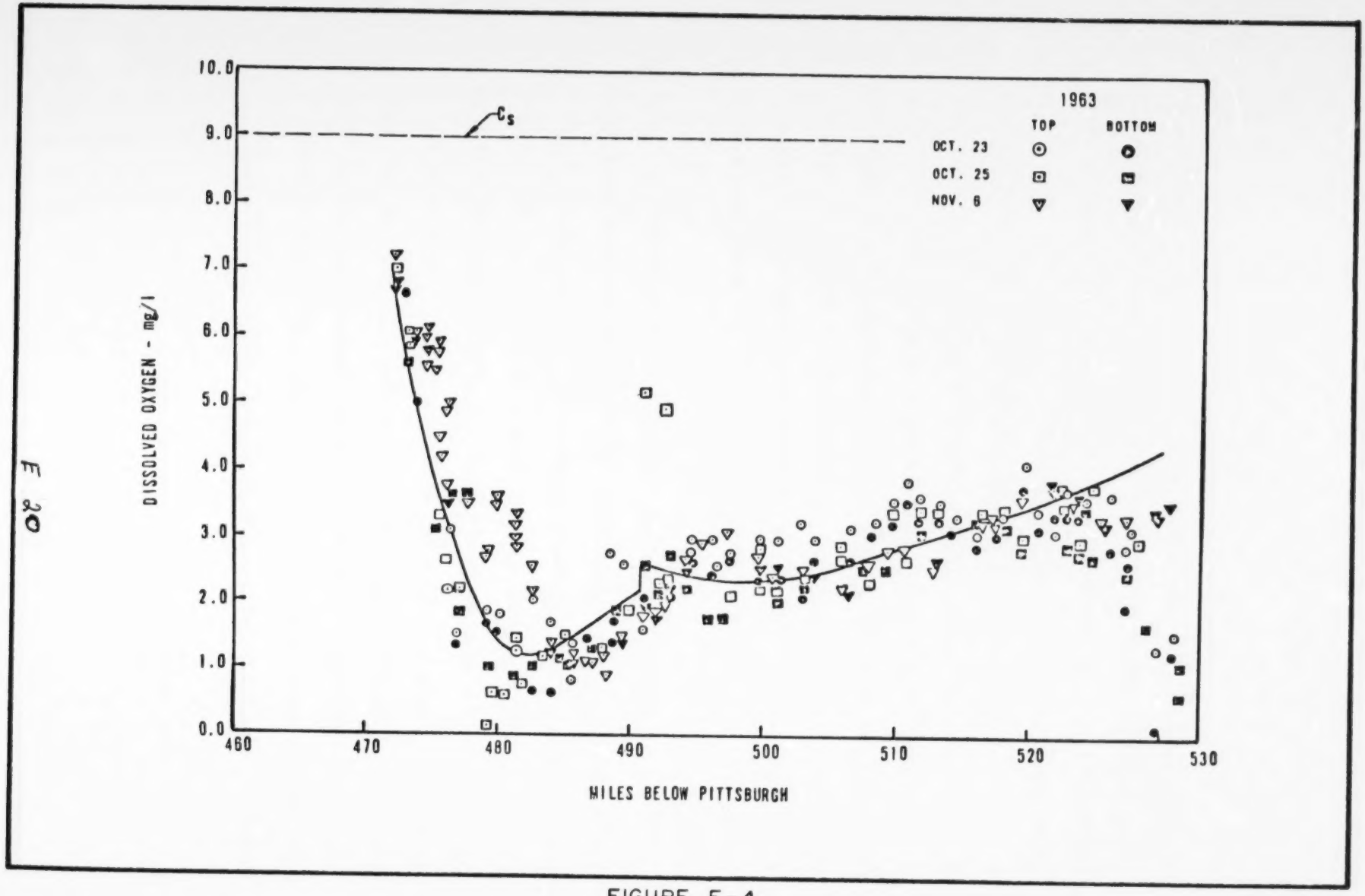

FIGURE E-4

OHIO RIVER

CALCULATED AND OBSERVED DO DISTRIBUTIONS

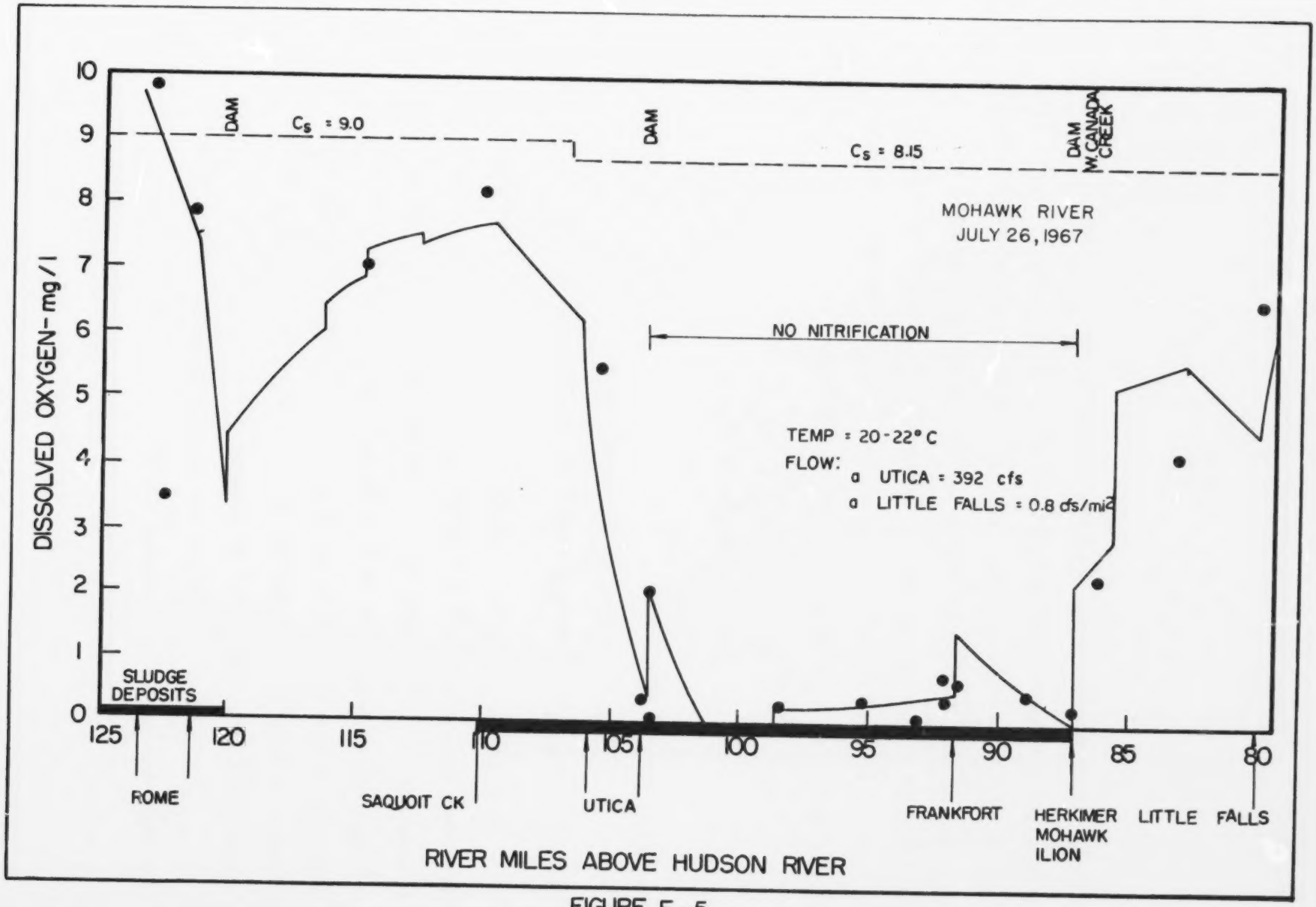

FIGURE E-5
DISSOLVED OXYGEN PROFILE
ROME TO LITTLE FALLS
MOHAWK RIVER

demand requires dissolved oxygen, while removal of the ultimate oxygen demand (organic matter) by settling does not directly use dissolved oxygen. As one proceeds downstream, the physical removal is completed and only oxidative removal remains. In such a situation, a semilog plot similar to Figure E-3 would probably yield two straight lines as shown in Figure E-6. The slopes of these lines are defined by:

$$\begin{aligned} &\text{Zone I:} \quad \text{slope} = -K_r/U \\ &\text{Zone II:} \quad \text{slope} = -K_d/U \end{aligned} \qquad \text{(E-3)}$$

where:

K_r = BOD removal coefficient (1/day)

K_d = deoxygentaiton coefficient (1/day)

In this guide, a constant deoxygenation coefficient has been assumed down the length of the water body. This assumption is considered valid, since the guide was meant to be applied to municipal effluents with at least secondary treatment. As a result, verification with the modeling procedures presented in this guide for the hypothetical problem presented above, may only be approximate.

A constant background dissolved oxygen deficit of 1.0 mg/l has been recommended in the guidelines. However, if

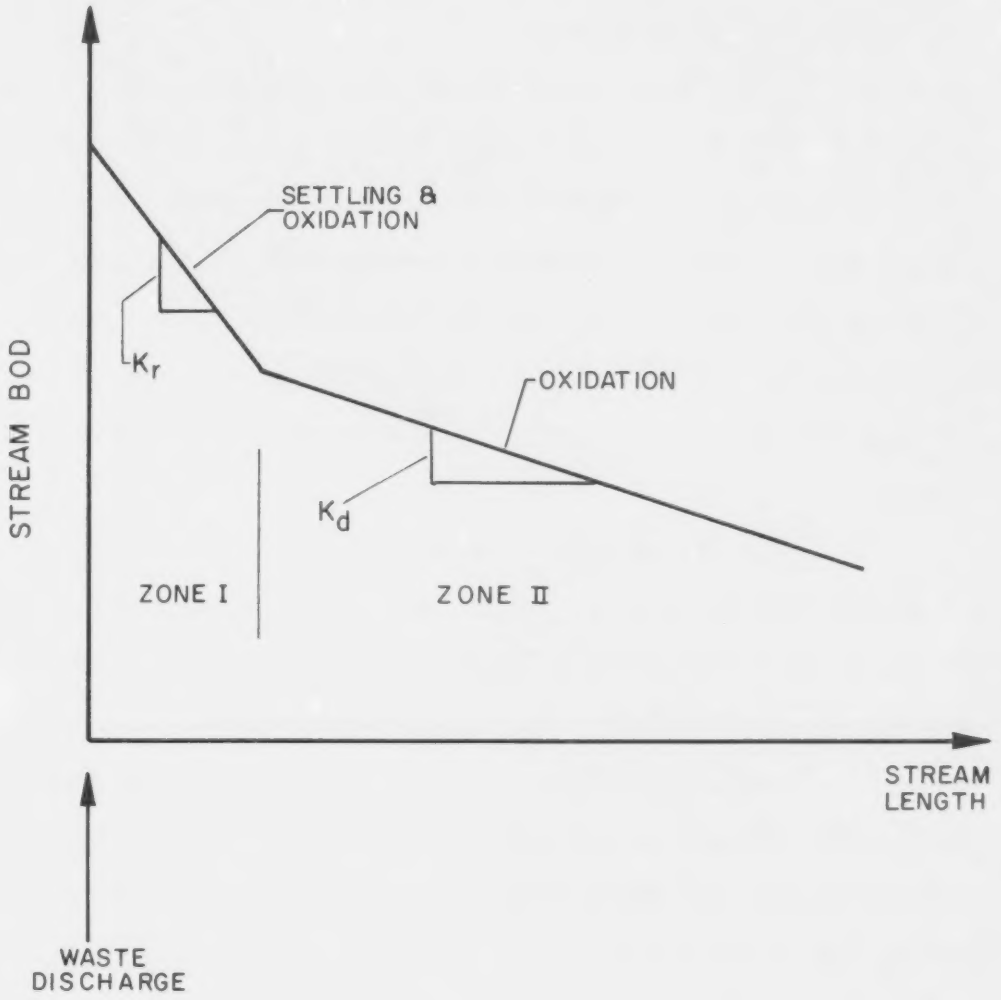

FIGURE E-6

ILLUSTRATION OF SPATIALLY VARYING BOD REMOVAL RATE

a verification analysis is being attempted where raw municipal effluents are involved, a background level of 1.5 to 2.0 is recommended due to the probable high organic content of the bottom due to settling.

It has been pointed out that nitrification is suppressed at low dissolved oxygen levels (<1.5 mg/l). This results in a delay in nitrification and a secondary dissolved oxygen sag is usually detected downstream. The guide procedures do not incorporate the calculation of such a secondary sag. However, if the primary dissolved oxygen sag is approximately verified, the coefficientts used may be considered valid.

High phytoplankton populations or extensive growths of rooted aquatic plants, will have a significant diurnal effect on the variation of dissolved oxygen. The modeling procedures do not incorporate this phenomenon.

Toxic substances (copper, arsenic, zinc, mercury, lead) will inhibit bacterial populations. Therefore, high concentrations of these toxicants will affect dissolved oxygen in the water body.

The inclusion of the above phenomena is of the realm of a detailed planning effort. Thomann[12] presents more sophisticated modeling procedures, which incorporate the modeling of these phenomena.

In summary, the constituents of the term ϕ may be evaluated from observed data. In situations where only dissolved oxygen data is available, a first approximation at verification should begin with the appropriate average values of K_a and K_d, or ϕ as presented in Figures A-2, A-3, and A-4 of Appendix A, respectively. These figures were developed on the basis of detailed planning efforts by Hydroscience and others. Final verification of observed data should be accomplished with coefficients within the ranges presented in these figures. If other phenomena above the assumptions used in this guide are present, good engineering judgment will determine the degree of validity of the coefficients obtained.

IV. Reaeration Over Dams

The reaeration occurring at dams is similar to the natural reaeration phenomenon occurring in a stream, and always drives the dissolved oxygen concentration of the water toward saturation.

Based on experimental data[13] and later verified with field data[14], the British developed the following equation for reaeration over dams:

$$r = 1 + 0.11\ ab(1 + 0.046T)H \qquad \text{(E-4)}$$

where:

r = deficit ratio $= \frac{C_s - C_a}{C_s - C_b} = \frac{D_a}{D_b}$

C_a = dissolved oxygen concentration above dam (mg/l)

C_b = dissolved oxygen concentration below dam (mg/l)

C_s = dissolved oxygen saturation (mg/l)

D_a = dissolved oxygen deficit above dam (mg/l)

D_b = dissolved oxygen deficit below dam (mg/l)

T = temperature (°C)

H = height (feet) through which the water falls

a = 1.25 in clear to slightly polluted water: 1.00 in polluted water: 0.80 in sewage effluents

b = 1.00 for weir with free fall: 1.3 for step weirs or cascades

Rearrangement of the above equation to the appropriate form to be used in this manual is as follows:

$$D_a - D_b = \left\{1 - \frac{1}{1 + 0.11\ ab(1 + 0.046T)H}\right\} D_a \qquad \text{(E-4a)}$$

An alternate equation developed by Mastropietro[15] from field data on the Mohawk River and Barge Canal in New York State is as follows:

$$\frac{C_b}{C_s} = (1 - 0.037H)\ \frac{C_a}{C_s} + 0.037H \qquad \text{(E-5)}$$

Rearrangement to the appropriate dissolved oxygen deficit formulation is:

$$D_a - D_b = 0.037H\ D_a \qquad \text{(E-5a)}$$

A plot of this formula which calculates the decrease in dissolved oxygen deficit over a dam, is presented in Figure E-7. This formulation was developed specifically for the Mohawk River and Barge Canal, and is valid for dams up to fifteen feet high and critical temperatures in the range of 20° to 25°C. Successful application of this formula was accomplished in a detailed planning study[11].

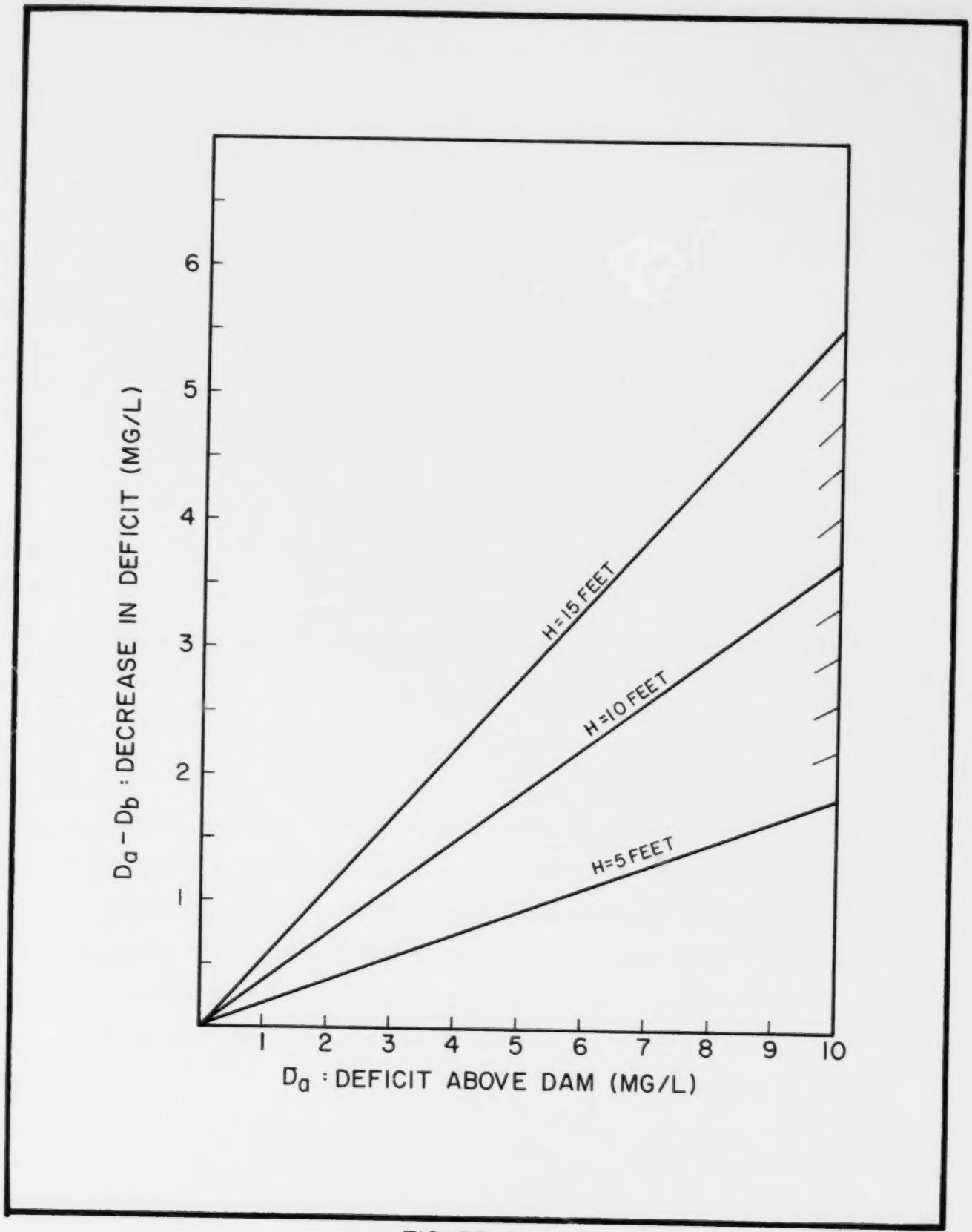

FIGURE E-7

REAERATION OVER DAMS

Under the same conditions, Equation (E-5a) predicts less reaeration than the British formulation and is, therefore, more conservative. In view of this and successful verification analyses in a detailed planning effort[11], it is recommended that Equation (E-5a) be used if conditions are within the stated dam height and temperature constraints.

To evaluate the effect of a dam, it is necessary to obtain the entire deficit profile even if there is only one waste source. The usual procedure in a stream analysis is to reevaluate the problem at the location of the dam. Reference is made to Equation (IV-5) of the main text (see page 72), repeated here as:

$$D = \frac{K_d L_o}{K_a - K_d} \left[e^{-\frac{K_d x}{U}} - e^{-\frac{K_a x}{U}} \right] \qquad \text{(E-6)}$$

This equation defines the deficit profile until the dam is reached. At this location, $X = X_d$, the ultimate oxygen demand not oxidized, L' is calculated from Equation (IV-2) of the main text (see page 69), repeated here as:

$$L = L_o e^{-\frac{K_d x}{U}} \qquad \text{(E-7)}$$

The remaining ultimate oxygen demand is analogous to a waste input at the dam. The deficit above the dam, D_a, calculated from Equation (E-6) at $X = X_d$ is applied to Equation (E-5a), which results in an estimate of the deficit, D_b, after dam reaeration. With these parameters, the deficit profile after the dam is defined by:

$$D = \frac{K_d L'}{K_a - K_d} \left[e^{-\frac{K_d}{U}(X - X_d)} - e^{-\frac{K_a}{U}(X - X_d)} \right] + D_b e^{-\frac{K_a}{U}(X - X_d)} \quad \text{(E-8)}$$

Due to the fact that these equations are linear, manipulation of Equation (E-8) results in the following equation:

$$D = \frac{K_d L_o}{K_a - K_d} \left[e^{-\frac{K_d x}{U}} - e^{-\frac{K_a x}{U}} \right] - (D_a - D_b)\, e^{\frac{K_a}{U}(X - X_d)} \quad \text{(E-9)}$$

Note that the first part of this equation is the same as if there were no dam in the river. Therefore, the procedures outlined for multiple waste sources (Section IV-D., pp.87), can be applied as originally presented. The analyst need only subtract the second term of Equation (E-9) from the total dissolved oxygen deficit response (Table IV-2, Column 8, pp.91) calculated.

Chart F shows the technique for computing the negative term of Equation (E-9). Figure F-1 is entered with mile point beginning at X = 0, at the dam location. With knowledge of K_a/U, this distance is converted to the dimensionless ratio of the negative deficit at the milepoint to the decrease in deficit at the dam. Since K_d, ϕ, and U have already been estimated, K_a/U is readily obtained.

Figure F-2 requires the decrease in deficit at the dam, $D_a - D_b$, due to dam reaeration. This value is obtained from Figure E-7, which is entered with the deficit previously calculated above the dam, and the height of the dam. The deficit values obtained from Figure F-2 are subtracted from column (8) of Table IV-2 at the appropriate milepoints, and the calculation of columns (9), (10), (11), and (12) proceed as before.

If one wishes to use the British formulation, the decrease in deficit at the dam can be directly calculated from Equation E-4a, and the same procedure as outlined above will be followed, except for use of Figure E-7.

Illustrative Example

The same problem as presented in Section IV-D-1-a (see page 95), will be reevaluated with a ten foot dam at

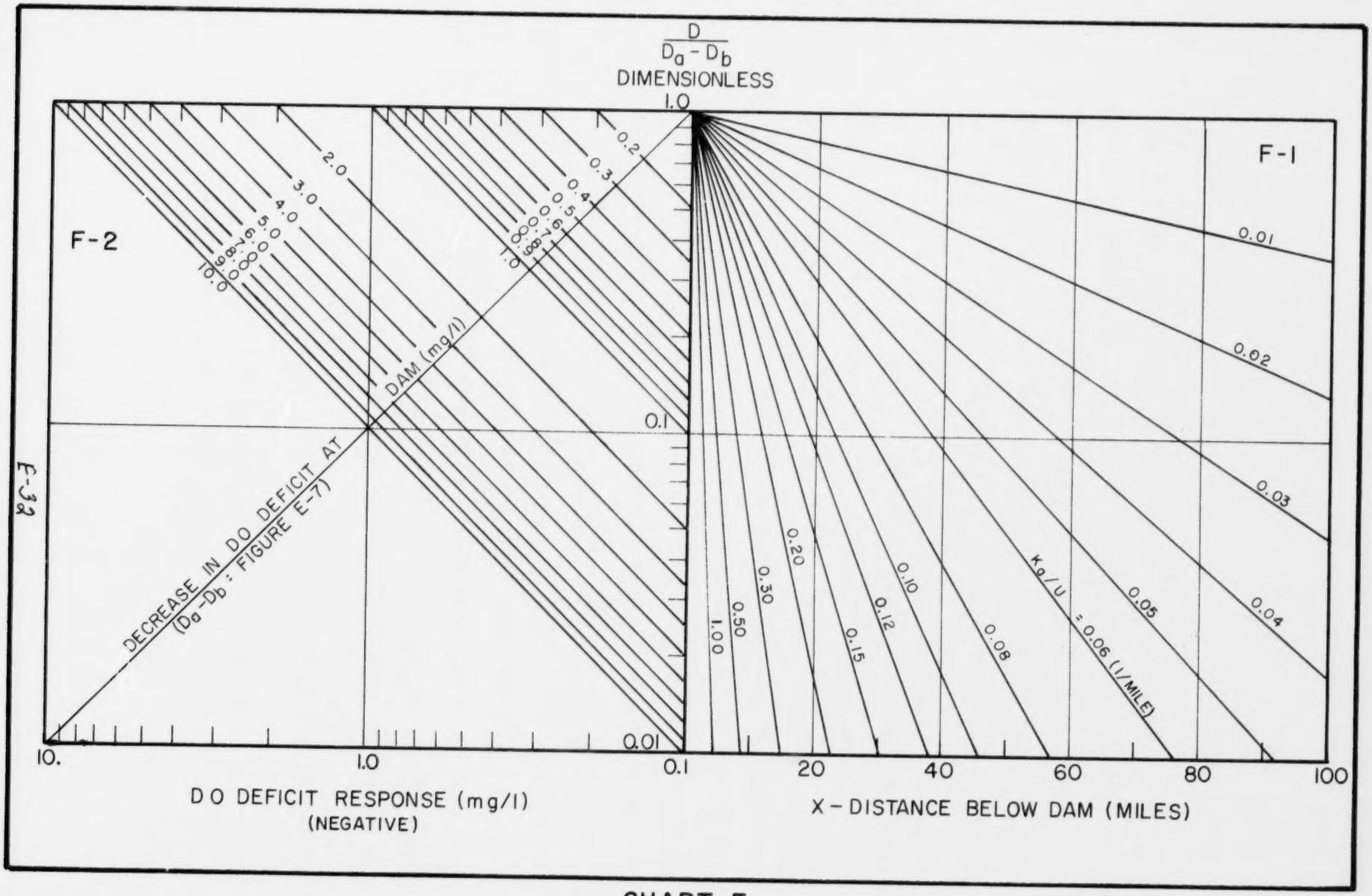

CHART F

EFFECT OF DAM ON DISSOLVED OXYGEN PROFILE

mile point 30. Table E-2, which is a continuation of Table IV-4 of the main text (see page 97), indicates the numerical analysis of the estimated dissolved oxygen profile due to the four waste sources with the dam included. Note that column (8) is the same as that of Table IV-4, except that it is relabeled Subtotal DO, etc. At milepoint 30, the Subtotal dissolved oxygen deficit response is 4.8 mg/l. From Figure E-7, with a dam height of 10 feet, the decrease in deficit at the dam is 1.8 mg/l. Column (8A) was obtained from Chart F using a K_a/U of 0.12/miles. Column (8B) was obtained from the subtraction of column (8A) from column (8), and represents the total dissolved oxygen deficit response. The remaining columns are obtained as previously outlined. Column (10) shows the estimated dissolved oxygen profile and indicates a new spatial minimum of 3.2. This may be compared to the spatial minimum of 2.6 previously calculated without the dam. As indicated, the standards are still violated from about mile 25 to mile 60.

TABLE E-2

ILLUSTRATIVE EXAMPLE
DISSOLVED OXYGEN – MULTIPLE WASTE SOURCE WITH DAM

(1) Total Downstream Distance (miles)	(8) Subtotal DO Deficit Response (mg/l)	(8A) Decrease In DO Deficit Due to Dam (mg/l)	(8B) Total DO Deficit Response (mg/l)	(9) DO Saturation (mg/l)	(10) D.O. (mg/l)	(11) DO Standard (mg/l)	(12) DO Standard Comparison (mg/l)
0	1.0		1.0	7.4	6.4	5.0	+ 1.4
5	1.8		1.8	7.4	5.6	5.0	+ 0.6
10	1.9		1.9	7.4	5.5	5.0	+ 0.5
15	1.9		1.9	7.4	5.5	5.0	+ 0.5
20	1.7		1.7	7.4	5.7	5.0	+ 0.7
25	4.2		4.2	7.4	3.2	5.0	- 1.8
30	4.8	1.8	3.0	7.4	4.4	5.0	- 0.6
35	4.5	1.0	3.5	7.4	3.9	5.0	- 1.1
38	4.0	0.7	3.3	7.4	4.1	5.0	- 0.8
40	4.1	0.5	3.6	7.4	3.8	5.0	- 1.2
46	3.7	0.3	3.4	7.4	4.0	5.0	- 1.0
50	3.7	0.2	3.5	7.4	3.9	5.0	- 1.1
55	3.3	0.1	3.2	7.4	4.2	5.0	- 0.8
60	2.8	-	2.8	7.4	4.6	5.0	- 0.4
70	2.0	-	2.0	7.4	5.4	5.0	+ 0.4
80	1.5	-	1.5	7.4	5.9	5.0	+ 0.9
90	1.3	-	1.3	7.4	6.1	5.0	+ 1.1

V. Temperature Coefficients

The deoxygenation (K_d) and reaeration (K_a) coefficients presented throughout this manual are for water temperatures of 20°C. Conversion to other temperatures can be made by applying the following formula:

$$K_T = K_{20}(\theta)^{T - 20} \qquad \text{(E-10)}$$

where K_T is the reaction coefficient at temperature, T (°C), K_{20} is the reaction coefficient at 20°C, and θ is a constant. The term θ is equal to $1.047^{(12)}$ and $1.024^{(12)}$ for the deoxygenation and reaeration coefficients, respectively. Nitrification is assumed to occur simultaneously, and at the same rate as carbonaceous oxidation. The θ value for nitrification is also assumed as 1.047. However, at low temperatures (<10°C) nitrification may be assumed suppressed.

VI. Evaluation of the Dispersion Coefficient, E

The dispersion coefficient, E, for an estuary or tidal river may be evaluated for a particular net advective flow from the observed steady-state concentration profile of salinity or chlorides, provided that vertical stratification

of these constituents is not pronounced. The underlying equation that may be utilized is Equation III-3 in the main text (refer to page 38), repeated here as:

$$C = C_o e^{gx} \qquad X \leq 0$$
$$C = C_o e^{jx} \qquad X \geq 0 \qquad \text{(E-11)}$$

Considering salinity or chlorides as conservative, and beginning the model at the furthest point downstream (location of maximum concentration), Equation (E-11) becomes:

$$C = C_o e^{\frac{UX}{E}} \qquad X \leq 0 \qquad \text{(E-12a)}$$
$$C = C_o \qquad X \geq 0 \qquad \text{(E-12b)}$$

where:

C	=	salinity or chloride concentration (mg/l)
C_o	=	maximum concentration at X = 0 (mg/l)
U	=	net advective velocity (miles/day)
E	=	dispersion coefficient (miles^2/day)
X	=	distance upstream (miles) - negative (usually, X = 0 at mouth of estuary)

Taking the natural log of Equation (E-12a) yields:

$$\ln C = \frac{U}{E} X + \ln C_o \qquad \text{(E-13)}$$

Therefore, a semi-logarithmic plot of log salinity or log chlorides versus distance upstream should yield a straight line, the slope of which is U/E. The dispersion coefficient, E, may then be calculated directly from the net advective velocity, which may be estimated from the freshwater inflow, and cross-sectional area of the estuary. Figure E-8 illustrates the computation for data collected by U.S. Geological Survey[16] for a reach of the Hudson River Estuary. Note that care must be taken to include the conversion from base 10 logarithms as given in semi-log plots to base e logarithms, as required by Equation (12a).

If vertical stratification of salinity or chlorides is encountered, it is suggested that the average over the depth be used, which will result in a first cut evaluation of the dispersion coefficient.

Dye data may also be used to obtain an estimate of dispersion coefficients. Reference is made to O'Connor[17] and Diachishin[18], for the appropriate procedures which are based on a more complicated analysis.

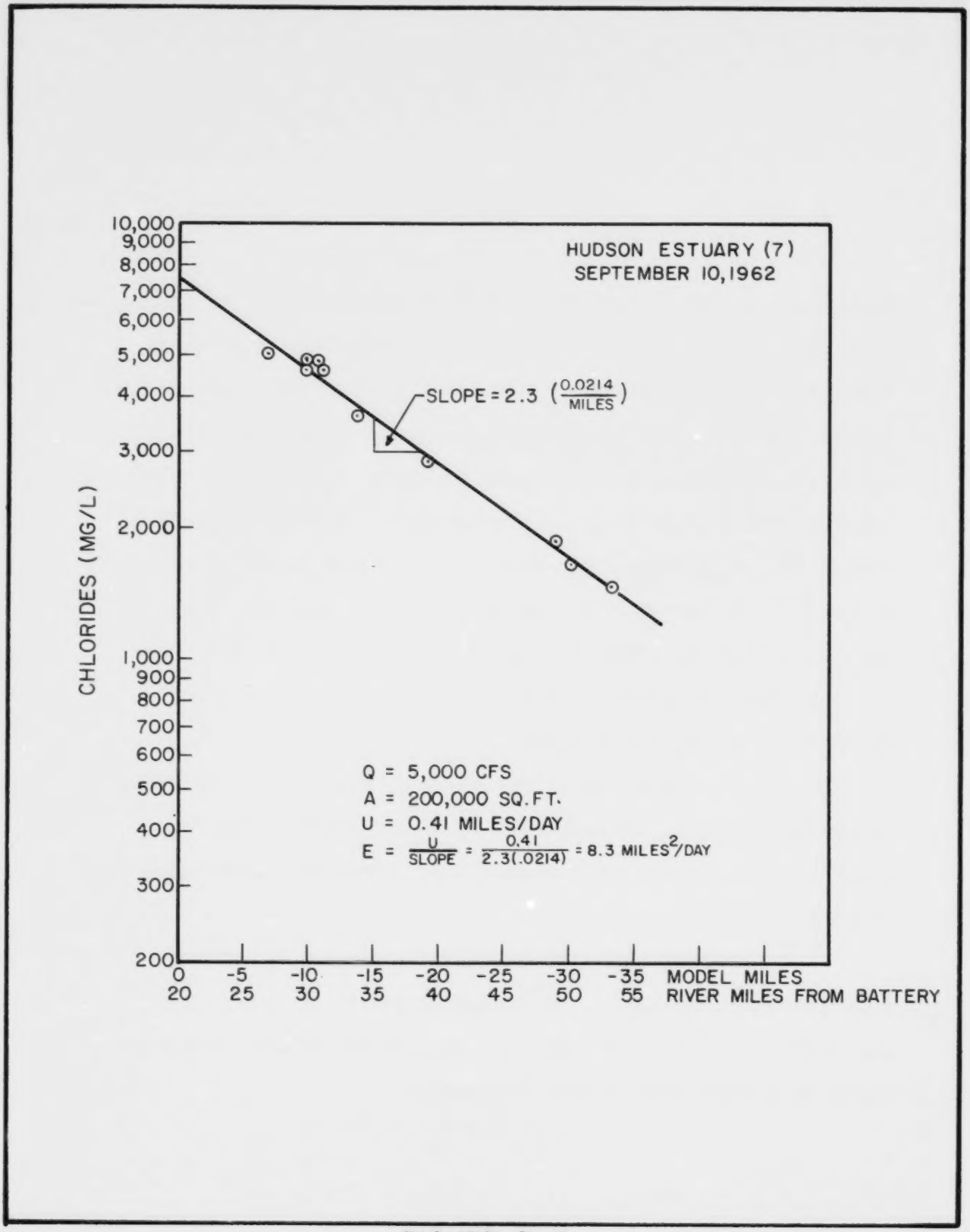

FIGURE E-8

ILLUSTRATION OF COMPUTATION OF E FROM CHLORIDE DATA

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Page	Line	Correction
52	Last line	e) .044 pounds nitrogen...
53	Table III-3 Last Column	
	Treatment Level 4	.05
	Treatment Level 5	.01
54	Table III-4 Last Column	
	Treatment Level 1	.040
	Treatment Level 2	.035
	Treatment Level 3	.035
87	Line 7	30°C, 5.9 mg/l...
99	Line 3	...minimum of about 2.6 mg/l
113	Line 5	(V-5)
124	Last Line	(V-6)
125	Line 7	...(by Equation V-6)...

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